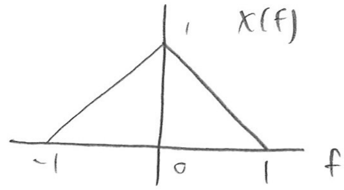
Final

1.a

$X(f) = \boxed{\Lambda(f)}$



1.b

$ESD = |X(f)|^2 = \boxed{\Lambda^2(f) = \begin{cases} (1-|f|)^2 & |f|<1 \\ 0 & \text{else} \end{cases}}$

1.c

$\text{Energy} = \int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} \Lambda^2(f)\, df = 2\int_0^1 (1-f)^2 df$

(even integrand)

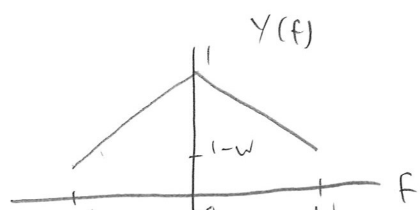
$= -2\frac{1}{3}(1-f)^3\Big]_{f=0}^{1} = \frac{2}{3} - 0 = \boxed{\frac{2}{3}}$

1.d.i

$E(W) = \int_{-W}^{W} |X(f)|^2 df = -\frac{2}{3}(1-f)^3\Big]_{f=0}^{W} = \frac{2}{3} - \frac{2}{3}(1-W)^3 = \frac{E}{2} = \frac{1}{3}$

$\frac{1}{3} = \frac{2}{3}(1-W)^3 \rightarrow \frac{1}{2} = (1-W)^3 \rightarrow \boxed{W = 1 - 2^{-\frac{1}{3}}} = 0.206$

1.d.ii



$Y(f) = \boxed{\Pi\left(\frac{f}{2W}\right)\Lambda(f)}$

$= (1-(1-W))\Lambda\left(\frac{f}{W}\right) + (1-W)\Pi\left(\frac{f}{2W}\right)$

$= \boxed{W\Lambda\left(\frac{f}{W}\right) + (1-W)\Pi\left(\frac{f}{2W}\right)}$

1.d.iii

$y(t) = \boxed{W^2 \operatorname{sinc}^2(Wt) + 2W(1-W)\operatorname{sinc}(2Wt)}$

Final

2.

$\delta(t)$ is the identity for convolution, so clearly $\hat{\delta}$ is the impulse response of the Hilbert transform: $\hat{\delta}(t) = \boxed{\frac{1}{\pi t}}$

Alternatively, the transfer function of the Hilbert transform is $-j\,\mathrm{sgn}(f)$, and $\mathcal{F}\{\delta(t)\} = 1$, so $\hat{\delta}(t) = \mathcal{F}^{-1}\{-j\,\mathrm{sgn}(f)\} = -j\frac{-1}{\pi j t} = \frac{1}{\pi t}$

3.a

$f_{LO} > f_c \rightarrow f_{LO} = f_c + f_{IF}$

$50 < f_c < 54 \rightarrow \boxed{57 < f_{LO} < 61 \text{ MHz}}$

3.b

Obviously, the desired range of f_c can be mixed to f_{IF}: $\boxed{50 < f_c < 54}$

Also, the image frequencies are mixed down to IF.

$|f_c' \pm f_{LO}| = f_{IF} \rightarrow f_c' \pm f_{LO} = \pm f_{IF}$

$$f_c' = \begin{cases} f_{IF} - f_{LO} \\ f_{IF} + f_{LO} \\ -f_{IF} + f_{LO} \\ -f_{IF} - f_{LO} \end{cases} = \begin{cases} -f_c \\ f_c + 2f_{IF} \\ f_c \\ -f_c - 2f_{IF} \end{cases}$$

$f_{LO} = f_c + f_{IF}$

$f_c' = f_c + 2f_{IF}$

$\boxed{64 < f_c' < 68}$

4.a

$\boxed{f_i(t) = f_c + f_\Delta m(t)}$ $\left(m_p = \max_t |m(t)| = 1\right)$

4.b

$x(t) = \cos(\theta(t))$, $\theta(t) = 2\pi \int f_i(t)\,dt = 2\pi f_c t + 2\pi f_\Delta \frac{1}{2\pi f_m}\sin(2\pi f_m t)$

$x(t) = \boxed{\cos\left(2\pi f_c t + \frac{f_\Delta}{f_m}\sin(2\pi f_m t)\right)}$

4.c

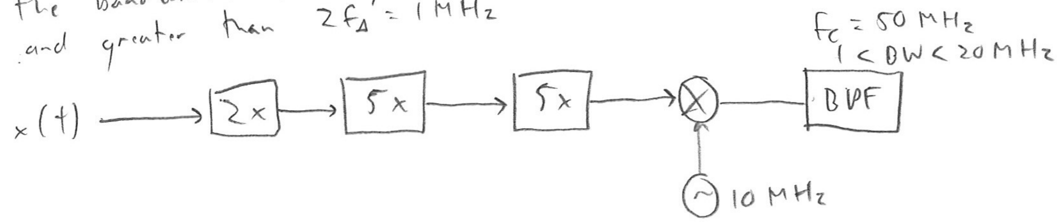
using Carson's rule, $BW \approx 2f_\Delta + W$. Clearly, $W = f_m$ because $m(t)$ is sinusoidal. $\boxed{BW \approx 20\text{ kHz} + f_m}$

Final

4.d)

We need $f_\Delta' = 500\text{ kHz} = 50 \cdot f_\Delta = 2 \cdot 5 \cdot 5 f_\Delta$

$1 \cdot 50 = 40 + 10$, so we need to shift by $+10$ MHz after the frequency multipliers,
the bandwidth of the BPF must be less than $50 - (40 - 10) = 20$ MHz
and greater than $2 f_\Delta' = 1$ MHz



5.a)

- FM has better noise immunity than AM. It can be amplified by more efficient non-linear amplifiers. Because of the constant envelope, the power is constant;
- FM requires more bandwidth than an equivalent AM signal. The receivers are more complicated (at least compared to DSB AM), though this is not a major issue in practice.

5.b)

- Antennas are only efficient if they are sized to within a couple of orders of magnitude of the wavelength they are receiving. This means higher frequencies result in smaller efficient antennas, but many signals (speech, music, low bandwidth data) are inherently low-frequency, so we shift them up.
- We can multiplex many separate signals on a shared medium by shifting them to different carrier frequencies (FDM).
- We can avoid interference at a particular frequency by shifting the signal out of that band.

6)

Because x is WSS, its PSD is well-defined, and we know how it transforms as it goes through an LTI system (which the Hilbert transform is).

$$G_{\hat{x}}(f) = |H(f)|^2 G_x(f) = |-j\,\mathrm{sgn}(f)|^2 G_x(f) = \boxed{1 \cdot G_x(f)} \quad \text{(unchanged)}$$